16 January, 1997

Variable Field Bending Magnets for Recirculating Linacs

B. J. King

email: Bruce.King@cern.ch

Abstract

A lattice of single aperture superconducting variable field bending magnets is proposed as a cheap and practical way to recirculate the beams in recirculating linear accelerators. It is shown that the VFBM's can be configured to provide strong focusing in both transverse planes for the full range of beam momenta transported by the lattice.

1 Introduction

Recirculating linacs, such as that at CEBAF and those proposed for future muon colliders [1], economise on RF cavities by circulating the beam through them several times, at increasing beam energies. This scheme transfers a large fraction of the cost of acceleration to the magnet lattices which bend the beam around to return it to the RF cavities, so it is important to construct the bending lattice as cheaply as possible.

A lattice of conventional superconducting dipole magnets, such as used in storage rings, cannot be used for recirculating linacs, since they cannot be ramped quickly enough to keep up with the increase in beam energy in successive passes through the arcs. Existing schemes for the lattice include [1] multiple aperture superconducting magnets, such as used at CEBAF, and interspersing single aperture superconducting magnets with fast ramping warm magnets. However, both of these options are expected to be relatively expensive compared to single aperture superconducting magnets.

This note introduces the idea of a lattice of single aperture superconducting variable field bending magnets (VFBM's). The point of using a variable field is that successive passes of the beam, at increasing beam energies, can be deflected by equal amounts by steering the beam to progressively higher field regions of the apertures. It will be shown that such magnets can be arranged in a bending lattice which is strongly focusing in both transverse views.

A local coordinate frame for the magnets will be used such that the beam travels in the z direction, the bend direction is horizontal and along the x direction, and the y coordinate gives the vertical displacement.

Throughout the paper, small angle approximations will be used for the deflection of the beam in each magnet. Also, it will be assumed that each magnet has a constant field along the z direction and end effects due to the finite lengths of the magnets will be neglected. These approximations should not affect the general

validity of the concept. Unless otherwise specified, magnetic fields are given in units of Tesla, lengths in meters, currents in Amperes and beam momenta in units of GeV/c.

The note is organised as follows. The following section gives a general discription of the magnetic fields in VFBM's and their focusing properties, and introduces the concept of a strong focusing lattice of VFBM's with alternating gradients, in close analogy to the conventional strong focusing lattice using quadrupole magnets. It is noted that the strong focusing property will apply to all momenta if the magnet focal length can be made independent of beam momentum. Section 3 gives an explicit prescription for doing this, by an appropriate choice of beam trajectories and field distributions. Section 4 addresses the question of how to design the coil configuration to produce the desired VFBM fields and section 5 provides an illustrative set of values for the magnet and lattice parameters. Further studies that will be needed to further assess the idea of a VFBM lattice are outlined in section 6, before summarizing the note in the final section.

2 Overview of Magnets and Lattice

The VFBM's are assumed to have a field which is independent of the coordinate along the beam,

$$\vec{B} \equiv \vec{B}(x, y), \tag{1}$$

and there is no field component along the beam:

$$B_z \equiv 0. \tag{2}$$

For all beam energies, the beam center will always be assumed to pass through the x-axis, i.e. y=0, which is a symmetry axis for the magnetic field and which will be referred to as the "beam plane". This implies that the field is vertical in the bend plane,

$$\vec{B}(x, y = 0) \equiv B_y(x)\hat{y}, \tag{3}$$

and the horizontal component of the field is identically zero:

$$B_x(x, y = 0) \equiv 0. \tag{4}$$

A VFBM of length l will bend a beam of momentum p and position x through an angle

$$\theta(x, p) = \frac{0.3B_y(x)l}{p}. \tag{5}$$

In addition, since the field gradient along the x axis is non-zero the beam will be either focused or defocused in the bend plane with a focal length, f, given by:

$$f = \pm\frac{p}{0.3Gl}, \tag{6}$$

where

$$G \equiv \frac{\partial B_y}{\partial x}. \tag{7}$$

The plus and minus signs in equation 6 imply that the beam is focused or defocused in the bend plane, respectively. Which of the two cases applies depends on the beam charge sign and whether it is travelling in the positive or negative z direction. For example, if the choice is made such that the bend direction is in the positive x direction then it is clear that a positive field gradient is defocusing (the field is stronger on the inside of the bend) and a negative gradient is focusing.

The gradients of the magnetic field in the beam plane (y=0) are constrained by Maxwell's equations in vacuo. The vanishing of the divergence of B implies that

$$\frac{\partial B_y}{\partial y} = -\frac{\partial B_x}{\partial x}. \tag{8}$$

This is identically zero in the bend plane, from equation 4, which simply means that the bending field remains constant to first order above and below the bend plane.

Since the field is independent of z, the x- and y-components of Maxwell's curl equation are trivially zero. More important is the z-component of the curl equation:

$$\frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x}. \tag{9}$$

This is easily seen to imply that the focal length in the vertical (y) direction will have equal magnitude and opposite sign to that in the horizontal (x) direction. In other words, if the magnet is focusing with a given strength in the x direction then it will be defocusing with equal strength in the y direction, and vice versa.

Of course, this focusing property is exactly the same as in a quadrupole magnet, so a lattice of VFBM's will have exactly the same focusing properties as a quadrupole lattice with the same magnet focal lengths. In particular, it is obvious that a lattice of VFBM's with equal magnetic fields and alternating gradients at the aperture center, (x=0, y=0), can be arranged to produce strong focusing in both planes for a beam of some chosen momentum passing through the aperture centers, exactly as is normally done with quadrupole magnets. We will refer to this chosen momentum as the central momentum, p_0.

The main point of this paper is to demonstrate that the strong focusing behaviour at the central momentum can be applied identically to all the beam momenta passing through the magnet lattice. This is achieved by arranging the beam trajectories and variation of the on-axis magnetic field such that the focal lengths for all momenta are equal to those at p_0, thus ensuring that the focusing properties are identical. An explicit prescription for doing this forms the topic of the following section.

3 Calculating Beam Trajectories and On-Axis Magnetic Fields

The goal of this section is to find the magnetic field variation in the VFBM's and the beam positions, $x^{\pm}(p)$, which will give the correct bend angle in the lattice along with optimal strong focusing of the beam in both views. The superscript "$\pm$" in the $x^{\pm}(p)$ distinguishes between the two types of VFBM's used in the lattice: the "plus" refers to the magnets with the positive on-axis field gradient in the positive x direction and the "minus" to those magnets with negative field gradient in the

x-view.

For definiteness, we assume the bend direction to be towards the positive x-axis, in which case the plus magnets are defocusing in the bend plane and the minus magnets are focusing, as already mentioned.

The beam position can be solved order-by-order by expanding in a Taylor series about the central momentum, p_0. In this section we will only solve for the first order terms:

$$x^{\pm}(p) = K^{\pm}.(p - p_0) + ..., \tag{10}$$

where K^+ and K^- are constants for the plus and minus magnets, respectively, and both zeroth order terms vanish due to the definition of p_0 and the choice of coordinate system:

$$x^{\pm}(p_0) \equiv 0. \tag{11}$$

The field-times-length of each of the magnets is conveniently defined for MKSA units as:

$$b(x) \equiv 0.3B_y(x, y = 0)l. \tag{12}$$

This definition is chosen so the bend angle of the magnet, equation 5, takes the simple form:

$$\theta(x, p) = \frac{b(x)}{p}. \tag{13}$$

The field-times-length can also be expanded in a Taylor series:

$$b^{\pm}(x) = b_0 \pm g_0.x \pm g_1^{\pm}.\frac{x^2}{2} + ... \tag{14}$$

In this equation the x values are understood to be those of the beams at the given momentum, $x = x^{\pm}(p)$, and we have used the assumptions that the bending fields at the central momentum, p_0, are equal and the field gradients are equal but with opposite signs. The derivative of equation 14 gives the Taylor expansions for the field gradients:

$$\frac{\partial b^{\pm}(x)}{\partial x} \equiv g^{\pm}(x) = \pm g_0 \pm g_1^{\pm}.x + ... \tag{15}$$

It is seen that equation 6, for the magnet focal lengths in the bend plane, can be written as:

$$f^{\pm} = \mp \frac{p}{g^{\pm}}. \tag{16}$$

Since strong focusing would be expected to be most effective for focal lengths roughly equal to the spacing between magnet centers, L, it simplifies the equations (but is not necessary) to assume that this condition holds exactly, i.e.:

$$f^{\pm} \equiv L \equiv \frac{p_0}{g_0}. \tag{17}$$

These equivalences use equation 16 and the key assumption that the focal length can be chosen to be independent of beam momentum.

We are now ready to derive the displacement constants K^+ and K^-. For non-zero $x^{\pm}$ the bend angles in the plus and minus magnets, $\theta^{\pm}(p)$ won't be equal to the central bend angle,

$$\theta_0 = \frac{b_0}{p_0}. \tag{18}$$

Instead, they are easily seen to be given by:

$$\theta^{\pm}(p) = \theta_0 \pm 2\frac{x^+(p) - x^-(p)}{L}. \tag{19}$$

(The average of the two bend angles is equal to θ_0, as it must be.)

On expanding these equation 19 to first order in $x^{\pm}$ and $p - p_0$, using equations 10, 14, 15, 16 and 17, and solving for K^+ and K^- one easily obtains:

$$K^+ = 5\frac{b_0}{g_0 p_0} \tag{20}$$

and

$$K^- = 3\frac{b_0}{g_0 p_0}. \tag{21}$$

From the definitions of K^+ and K^-, equation 10, this can be rewritten in terms of the displacements:

$$x^+(p) = 5\frac{b_0}{g_0}\frac{p - p_0}{p_0} \tag{22}$$

and

$$x^{-}(p) = 3\frac{b_0}{g_0}\frac{p-p_0}{p_0}. \tag{23}$$

This is the desired first order approximation to the beam trajectories for beam momenta different from the central momentum, p_0.

Now that the beam trajectories are known to first order it is possible to apply the strong focusing assumption of equation 17 to obtain the first order change in the field gradient. From equation 16 we obtain:

$$\frac{p_0}{g_0} = \frac{p}{g_0 + g_1^{\pm}.K^{\pm}(p-p_0) + ...}. \tag{24}$$

Solving these equations to the lowest nontrivial order gives, after some algebra:

$$g_1^{+} = \frac{g_0^2}{5b_0} \tag{25}$$

and

$$g_1^{-} = \frac{g_0^2}{3b_0}. \tag{26}$$

The first order coefficients in the gradient of the field that have just been derived are also, of course, the second order coefficients in the field itself. Since the first order derivation of the displacements $x^{\pm}(p)$ needed only the first order coefficients in the field it is clear that the second order field coefficients can be used to derive the second order correction to the displacements. In turn, the second order displacement coefficients will permit the derivation of the third order field coefficients, and so on.

In summary, alternate applications of the constraints on the bending fields and on the focal lengths enable the Taylor expansion coeffients of the magnetic field and the beam positions to be determined to arbitrarily high orders. Hence, the ideal beam positions for all momenta and the ideal on-axis magnetic field throughout the magnets can, in principle, be predicted to arbitrary accuracy.

Since the strong focusing principle would be expected to work for a range of focal lengths about the optimal value, albeit less effectively, our requirement that that focal length takes the optimal value for all momenta is unnecessarily strict. In

practice, the "rigorous" solutions for $x^{\pm}(p)$ and $b^{\pm}(x)$ obtained using the method outlined in this section could be used as a starting point for design iterations which might compromise the strong focusing power of the lattice for some momenta in order to improve on other features of the magnet design.

4 Layout of Magnet Coils

The preceding section specifies a magnetic field distribution, $B_y(x, y = 0)$, which is is smooth and monotonically varying but which cannot be expressed in closed form. Obviously, the current distribution to produce this field must be obtained by numerical means. This section describes a general minimization procedure to obtain a suitable layout for the conducting coils, and illustrates the method using a simple example.

In general, the desired magnetic field along the x axis will be produced by a 2-dimensional current distribution around the magnet aperture, $J(x, y)$, which is symmetric under reflection in the x-axis:

$$J(x, y) \equiv J(x, -y). \tag{27}$$

This current distribution will produce a magnetic field on the x-axis with zero horizontal component, $B_x(y = 0) \equiv 0$, and a vertical component given by:

$$B(x) \equiv B_y(x, y = 0) = 10^{-7} \int dx'dy' J(x', y') \frac{(x - x')}{(x - x')^2 + y'^2}, \tag{28}$$

using MKSA units.

The goal is to obtain an appropriate current distribution which gives an on-axis magnetic field closely approximating the desired field, $B^{true}(x)$. This can be achieved in the following steps:

1. specify the regions which can contain conductor and parameterize a sensible current distribution in these regions in terms of a small number of adjustable

parameters, C_i:

$$J(x, y) = J(x, y; C_i),\ i = 1, n. \tag{29}$$

2. define an error function to quantify the deviation of the on-axis field produced by the current distribution from the desired field. An appropriate error function is:

$$E[C_i] = \int dx(\frac{B^{true}(x) - B(x)}{B^{true}(x)})^2/(x_{max} - x_{min}), \tag{30}$$

   where the magnetic field $B(x)$ is given by equation 28 with the current distribution specified by the values of the C_i.

3. vary the C_i to minimize the error function.

To illustrate and test the procedure, an explicit current distribution was derived for the following simple case:

- an exponentially varying magnetic field along the x-axis: $B(x) = e^x$, for x in the range -1 to 1.
- B field from surface current along wedge-shaped magnet aperture.
- no requirement that the current sum to zero. This is equivalent to assuming that the excess current is returned at a very large distance from the aperture.

In more detail, the surface current, $K(x, y(x))$ was parameterized to have a quadratic form:

$$\begin{aligned} K(x, y(x)) &= C_1 + C_2.x + C_3.(2x^2 - 4), \quad \text{for } -2 < x < 2 \\ &= 0, \qquad\qquad \text{otherwise.} \end{aligned}$$

The two y coordinates of the current for each x, symmetric about the x axis, were specified by a linear form with one free parameter and a minimum aperture of 0.1 units at $x = -2$:

$$y(x) = \pm[0.1 + C_4.(x + 2)]. \tag{31}$$

The MINUIT minimalization software package [2] was used to find the values of the C_i which minimized the error function of equation 30. Numerical integrations were used to evaluate the error function and the on-axis magnetic field of equation 28. The constant factor in front of the magnetic field equation was neglected, corresponding to an overall scale factor in the magnetic field strength.

The optimal current distribution was obtained for the parameter values:

$$C_1 = 7.228; \quad C_2 = 2.073; \quad C_3 = 1.107; \quad C_4 = 0.656. \tag{32}$$

Figure 1 displays the x distribution of this current and figure 2 illustrates the level of agreement between the resulting magnetic field and the exponential distribution. The root mean square deviation of the on-axis magnetic field from the desired exponential form, given by the square root of the error function, was found to be 1.9% for the region between $x = -1$ and $x = 1$.

It is clear that the procedure can be modified to work for a more realistic magnet design. It is also obvious that infinitely many conductor configurations can be chosen to produce an acceptably good approximation to the desired field along the symmetry axis of the magnet. The decision between possible configurations can therefore be based on other factors, such as good field quality off-axis, simplicity of production, a desirable aperture shape, cheap cost and good mechanical properties.

5 Example Lattice Parameters

Table 1 gives an illustrative example set of parameters for the VFBM lattice of the final recirculating linac in a muon collider with a centre of mass collision energy of about 4 TeV. The values of these parameters should not be taken too seriously. They have not been optimized or particularly carefully chosen and their only purpose is to give a rough feel for the parameter values that might be expected for a more realistic lattice.

The first 4 parameters in the table, p_0, B_0, G_0 and l, essentially define the lattice at the central momentum value. The next three parameters, f, L and R, follow from relations given in the preceding sections.

For a beam with position divergence $< x >$ and angular divergence $< \phi >$ it is assumed that a strong focusing lattice of focal length f will have a maximum 1-sigma beam envelope, S of order:

$$S \sim < x > + < \phi > f. \tag{33}$$

(And similarly for the y coordinate.) If the phase space in the x view,

$$P_x \equiv < x >< \phi >, \tag{34}$$

is assumed to be fixed independent of the values of the two terms, $< x >$ and $< \phi >$, then,

$$S \sim < x > + \frac{P_x}{< x >} f \tag{35}$$

and this is minimized for

$$< x > \sim \sqrt{P_x . f}, \tag{36}$$

at a value of

$$S \sim 2\sqrt{P_x . f}. \tag{37}$$

This gives a numerical value of 0.6 mm at 1 TeV and using the phase space size, $P_x \sim 10^{-8}$ m.rad, of the same order as assumed in reference [1].

The maximum and minimum momenta accepted by the lattice have somewhat arbitrarily been assumed to be factors of two greater than and less than p_0, respectively. The average bending field needed for p_{max} is therefore twice as big as that for p_0. Presumably, almost all of the bending power will come from the “plus” magnets, requiring another factor of two stronger field in these magnets. Hence the maximum field, B_{max} might be roughly four times larger than B_0, and the minimum field close to zero. The maximum and minimum gradients, G_{max} and G_{min}, follow

from the central field and gradient by scaling in proportion to the momentum using equation 17.

The height, $Y_{aperture}$, of the aperture at the central x value, x=0, was chosen to be about 30 sigma wide at the maximum beam size, S_0. The width of the aperture in x, $X_{aperture}$, can be estimated simply by the dimensional argument of dividing the maximum magnetic field by the central gradient:

$$X_{aperture} \sim \frac{B_{max}}{G_0}. \tag{38}$$

6 Outlook

The following studies still need to be undertaken to confirm that strong focusing VFBM lattices are feasible and practical for recirculating linacs:

1. continue to higher order the Taylor series expansion of the on-axis magnetic field of the VFBM's. This will provide a better estimate of the range of beam momenta which can be accepted by a VFBM lattice.

2. use the procedure of section 4 to determine a realistic and appropriate magnet coil configuration that will produce the desired on-axis magnetic fields.

3. perform computer-based ray-tracing simulations of a beam through a VFBM lattice, to check that it performs as expected.

If the bending lattice performs as hoped then it will still need to be matched to the linacs for each pass of the beam, in beam position and direction and in the phase of the RF cavities. This could possibly be done using a dispersive section of superconducting magnets or, if this is found to be impractical, by using fast ramping warm magnets.

parameter	value
central momentum, p_0	1 TeV/c
central field, B_0	1.5 T
central field gradient, G_0	40 T/m
magnet length, l	8 m
focal length, f	10 m
lattice spacing, L	10 m
bending radius of lattice, R	2.8 km
maximum beam size at p_0, S	0.6 mm
maximum momentum, p_{max}	2 TeV/c
minimum momentum, p_{min}	0.5 TeV/c
maximum field, B_{max}	6 T
minimum field, B_{min}	0 T
maximum gradient, G_{max}	80 T/m
minimum gradient, G_{min}	20 T/m
aperture height, $Y_{aperture}$	2 cm
aperture width, $X_{aperture}$	15 cm

Table 1: **Example parameters for the VFBM lattice of the final recirculating linac in a muon collider with a centre of mass collision energy of about 4 TeV. See text for further details.**

7 Conclusions

The idea of using strongly focusing lattices of VFBM's in recirculating linear accelerators has been found to be quite promising and worthy of further study.

References

[1] The Muon Muon Collider Collaboration,
Muon Muon Collider: a Feasibility Study.
BNL-52503, Fermi Lab-Conf.-96/092, LBNL-38946.

[2] The MINUIT software package.
CERN Computer Centre Program Library.

Figure 1: The surface current distribution used to produce an approximately exponential bending field in the VFBM.

Figure 2: The bending field in the VFBM (solid line) produced by the surface current distribution of figure 1. The dashed curve is the "ideal" exponential field that the current distribution was tuned to reproduce.